TOPICS IN PROBABILITY THEORY AND STOCHASTIC PROCESSES
Home Work 4, due on THURSDAY SEPTEMBER 25,
Instructor: Prof. Artem Zvavitch

Problem 1. *Consider an organization of N employees (N is a HUGE number!). Each employee has one of three possible job classificatins and changes classifications (independently) according to a Markov chain with transition probabilities*

$$\mathbf{P} = \begin{pmatrix} .7 & .2 & .1 \\ .2 & .6 & .2 \\ .1 & .4 & .5 \end{pmatrix}.$$

What percentage of employees are in each classification?

Problem 2. *For the Markov chain with states $1, 2, 3, 4$ whose transitional probability*

$$\mathbf{P} = \begin{pmatrix} .4 & .2 & .1 & .3 \\ .1 & .5 & .2 & .2 \\ .3 & .4 & .2 & 0.1 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

find f_{i3} (probability that Markov chain EVER make transition from i to 3) and s_{i3} (expected number of times in state 3 , given that we started from i), for $i = 1, 2, 3$.

Problem 3. *In a branching process having $X_0 = 1$ and $\mu < 1$ show that the expected number of individuals ever exist in this population is given by $\mu/(1 - \mu)$. What of $X_0 = n$?*

Problem 4. *Please find π_0 for the branching processes when*

- $P_0 = 1/4, P_2 = 3/4.$
- $P_0 = 1/4, P_1 = 1/2, P_2 = 1/4.$
- $P_0 = 1/6, P_1 = 1/2, P_2 = 1/3.$