TOPICS IN PROBABILITY THEORY AND STOCHASTIC PROCESSES
Home Work 6, due on THURSDAY October 16,
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Problem 1. *let $N_1(t)$ and $N_2(t)$ (t is a time in hours) be two independent Poisson processes with rates λ_1 and λ_2, respectively, counting the number of customers arriving in stores 1 and 2, respectively.*

(1) *What is the probability that a customer arrives in store 1 before any customer arrives in store 2?*
(2) *What is the probability that in the first hour, a total of exactly four customers have arrived at the two stores?*
(3) *Given that exactly four customers have arrived at the two stores, what is the probability that all four went in store 1?*
(4) *Let T denote the time of arrival of the first customer at s tore 2. Then $N_1(T)$ is the number of customers in store 1 at the time of the first customer arrival at store 2. Find the probability distribution of $N_1(T)$ (i.e. for each k find $P(X(T) = k)$).*

Problem 2. *Potential customer arrive at a single-server station in accordance with a Poisson process with rate λ. However, if the arrival finds n customers already in the station, the he will enter the system with probability p_n (i.e. with probability $1 - p_n$ he will go away). Assuming an exponential service rate μ, set this up as a birth and death process and determine the birth and death rates.*

Problem 3. *Consider a birth and death process with birth rates $\lambda_i = (i + 1)\lambda$, and death rates $\mu_i = i\mu$, $i \geq 0$.*

(1) *Determine the expected time to go from state 0 to state 4.*
(2) *Determine the expected time to go from state 2 to state 5.*
(3) *Determine the variances in (1) and (2).*

Problem 4. *The birth and death process with parameters $\lambda_n = 0$ and $\mu_n = \mu$, $n > 0$ is called a pure death process. Find $P_{ij}(t)$.*