TOPICS IN PROBABILITY THEORY AND STOCHASTIC PROCESSES
Home Works 8, 9 due on THURSDAY December 4,
Instructor: Prof. Artem Zvavitch
This HW is 40 points

Problem 1. *Let $\{B(t), t \geq 0\}$ be a standard Brownian motion.*

- *What is the distribution of $B(s) + B(t)$, $s \leq t$.*
- *Compute the conditional distribution of $B(s)$ given that $B(t_1) = A$ and $B(t_2) = B$, where $0 < t_1 < s < t_2$ (i.e. find $\mathbb{P}(B(s) \leq x | B(t_1) = A, B(t_2) = B)$)*
- *Compute $\mathbb{E}[B(t_1)B(t_2)B(t_3)]$, for $t_1 < t_2 < t_3$.*

Problem 2. *If T_a is a heating time of a standard Brownian motion process. Find $\mathbb{P}(T_1 < T_{-1} < T_2)$.*

Problem 3. *Let $\{X(t), t \geq 0\}$ be a Brownian motion with drift coefficient μ and variance δ^2. What is conditional distribution of $X(t)$ given that $X(s) = c$ when $s < t$? when $s > t$?*

Problem 4. *A stock is presently selling at a price \$ 50 per share. After one time period, its selling price will (in present value dollars) be either \$ 150 or \$ 25. An option to purchase y units of the stock at time 1 can be purchased at cost cy. Use the Arbitrage Theorem to find c so that there will be NO sure win. Also take $c = 4$ and show how you can guarantee a sure win.*

Problem 5. *The current price of stock is* 100*. Suppose that the logarithm of the price of the stock changes according to a Brownian motion with drift coefficient $\mu = 2$ and variance parameter $\delta^2 = 1$. Give the Black-Scholes cost of an option to buy the stock at time* 10 *for the cost of* 100 *per unit?* 120 *per unit? Assume that the continuously compounded interest is 5 percent.*