

INTRODUCTION TO TOPOLOGY I
Home Work 1, due on THURSDAY SEPTEMBER 5,
Instructor: Prof. Artem Zvavitch

Problem 1. *If $f : A \rightarrow B$ and $B_0, B_1 \subset B$ prove that*

$$f^{-1}(B_0 - B_1) = f^{-1}(B_0) - f^{-1}(B_1).$$

Problem 2. *Construct a function f which is a bijection from $[0, 1)$ to $(0, 1)$.*

Problem 3. *Let $f : A \rightarrow B$ and $g : B \rightarrow C$. Assume that f and g are injective what can you say about $f \circ g$? Assume that f and g are surjective what can you say about $f \circ g$? Now if $f \circ g$ is surjective what can you say about f and g ? Finally, if $f \circ g$ is injective what can you say about f and g ?*

Problem 4. *Prove that if an ordered set A has the least upper bound property, then it has the greatest lower bound property.*

Problem 5. *if $A \times B$ is finite does it follow that A and B are finite ?*

Problem 6. *If A and B are finite show that the set of all functions $f : A \rightarrow B$ is finite. Knowing cardinality of A and B find the cardinality of the set of functions $f : A \rightarrow B$.*

Problem 7. *Let A be a countable set, and B is finite, what can you say about cardinality of the set of functions $f : A \rightarrow B$.*