INTRODUCTION TO TOPOLOGY I
Home Work 2, due on THURSDAY SEPTEMBER 18,
Instructor: Prof. Artem Zvavitch

Problem 1. *Both sets* $\{1,2\} \times \mathbb{N}$ *and* $\mathbb{N} \times \{1,2\}$ *are well-ordered in the dictionary order. Do they have the same order type?*

Problem 2. *Consider a linear vector space* V*. For* $A, B \subset V$ *we define* $A + B = \{x + y | x \in A, y \in B\}$*. Assume* A, B *are a countable sets, what can you say about cardinality of* $A + B$*?*

Problem 3. *A* **subbasis** S *for a topology on* X *is a collection of subsetrs of* X *whose union equals* X*. The* **topology generated by the subbasis** S *is defined to be the collection* $\mathbb{T}$ *of all unions of finite intersection of elements of* S*. Check that* $\mathbb{T}$ *is a topology.*

Problem 4. *Let* X *be a topological space; let* A *be a subset of* X*. Suppose that for each* $x \in A$ *there is an open set* U *containing* X *such that* $U \subset A$*. Show that* A *is open in* X*.*

Problem 5. *Consider set* $X = \{a, b, s\}$*. Please all possible (deferent) topologies on this set. How many of them? Compare them (for each pair determine whether they are comparable or not, and if so find which is finer)*

Problem 6. *Let* $\{\mathbb{T}_\alpha\}$ *be a family of topologies on* X*, show that* $\cap \mathbb{T}_\alpha$ *is a topology. What can you say about* $\cup \mathbb{T}_\alpha$*? Also, please, show that there is a unique smallest topology on* X *containing the collection* $\{\mathbb{T}_\alpha\}$*, and unique largest topology contained in all* $\{\mathbb{T}_\alpha\}$*.*

Problem 7. *Show that the following set is countable* $\mathbb{B} = \{(a, b) | a < b, a \text{ and } b \text{ are rational}\}$*. Show that* $\mathbb{B}$ *is a basis that generates the standard topology on* $\mathbb{R}$*. Compare it with topology generated by* $\mathbb{B}' = \{[a, b) | a < b, a \text{ and } b \text{ are rational}\}$*. Compare topology generated by* $\mathbb{B}'$ *with the lower limit topology on* $\mathbb{R}$ *(topology generated by sets* $[a, b)$*,* $a, b \in \mathbb{R}$*,* $a < b$*).*

Problem 8. *Show that the collection of sets*

$$\{(x_1, x_2) \times (y_1, y_2) \times (z_1, z_2) | x_1, x_2, y_1, y_2, z_1, z_2 \text{ are rational}\}$$

is a basis for $\mathbb{R}^3$*.*