

INTRODUCTION TO TOPOLOGY I
Home Work 3, due on THURSDAY SEPTEMBER 29,
Instructor: Prof. Artem Zvavitch

Problem 1. Assume $A, B, A_\alpha \subset X$. Prove the following

- If $A \subset B$ then $\overline{A} \subset \overline{B}$.
- $\overline{A \cup B} = \overline{A} \cup \overline{B}$.
- $\overline{\bigcup A_\alpha} \supset \bigcup \overline{A_\alpha}$, show that equality is not necessary true.

Problem 2. Show that every ordered topology is Hausdorff.

Problem 3. Let $A \subset X$ and $B \subset Y$. Show that in the space $X \times Y$

$$\overline{A \times B} = \overline{A} \times \overline{B}$$

Problem 4. Find a function $f : \mathbb{R} \rightarrow \mathbb{R}$, which is continuous at precisely one point.

Problem 5. Let Y be an ordered set in the ordered topology. Let $f, g : X \rightarrow Y$ be continuous

- Show that the set $\{x : f(x) \leq g(x)\}$ is closed in X .
- Let $h : X \rightarrow Y$ be the function $h(x) = \min\{f(x), g(x)\}$. Show that $h(x)$ is continuous.

Problem 6. Let $f : A \rightarrow B$ and $g : C \rightarrow D$ be continuous functions. Let us define a map $f \times g : A \times C \rightarrow B \times D$ by the equation

$$f \times g(a \times c) = f(a) \times g(c).$$

Show that $f \times g$ is continuous.

Problem 7. Prove that if each space X_α is a Hausdorff space then $\prod X_\alpha$ is a Hausdorff space in both the box and product topologies.

Problem 8. Given sequences (a_1, a_2, \dots) and (b_1, b_2, \dots) of real number with $a_i > 0$ for all i , define $h : \mathbb{R}^\omega \rightarrow \mathbb{R}^\omega$ as

$$h((x_1, x_2, \dots)) = (a_1x_1 + b_1, a_2x_2 + b_2, \dots).$$

Show that if \mathbb{R}^ω is given the product topology then h is a homeomorphism of \mathbb{R}^ω with itself. What happens if \mathbb{R}^ω is given the box topology?