

## INTRODUCTION TO TOPOLOGY I

Home Work 6, due on October 5.

Instructor: Prof. Artem Zvavitch

**Problem 1.** Show that  $\mathbb{R}_l$  is not metrizable.

**Problem 2.** Let  $f, g : X \rightarrow Y$  be continuous; assume that  $Y$  is Hausdorff. Show that  $\{x \mid f(x) = g(x)\}$  is closed in  $X$ .

**Problem 3.** Show that closed subspace of the normal space is normal.

**Problem 4.** Show that every locally compact Hausdorff space is regular.

**Problem 5.** Give a direct proof of the Urysohn lemma for a metric space  $(X, d)$  by setting

$$f(x) = \frac{d(x, A)}{d(x, A) + d(x, B)}$$

**Problem 6.** Let  $X$  be completely regular; let  $A$  and  $B$  be disjoint closed subsets of  $X$ . Show that if  $A$  is compact, there is a continuous function  $f : X \rightarrow [0, 1]$  such that  $f(A) = \{0\}$  and  $f(B) = \{1\}$ .

**Problem 7.** Give an example showing that a Hausdorff space with a countable basis need not be metrizable.